Measuring students’ algebra, trigonometry, and geometry skills on a differential calculus for engineering course

Medición de las habilidades algebraicas, trigonométricas y geométricas de los estudiantes en el curso de cálculo diferencial en ingeniería

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Abstract:

This research presents the results of a study that involved the construction and validation of a measuring instrument to evaluate the algebra, trigonometry, and geometry skills that university students possess when starting an engineering degree and which are critical for students to perform properly in calculus courses. The instrument was designed by faculty members from the field of mathematics, all of whom hold at least a master’s degree and have taught calculus in the past. The study comprised of 40 items and its quality analysis was based on data collected from 875 incoming first-year students during the 2020-2022 academic cycle. Data analysis showed that items with medium difficulty and high discrimination have the highest predictive coefficient and correspond mainly to the field of geometry, specifically the topics of the straight line, circumference, and the cal-

Revision accepted: 2021-03-07

This is the English version of an article originally printed in Spanish in issue 282 of the **revista española de pedagogía**. For this reason, the abbreviation EV has been added to the page numbers. Please, cite this article as follows: De las Fuentes-Lara, M., Aguilar-Salinas, W. E., Justo-López, A. C., & Iñiguez-Monroy, C. G. (2022). Medición de las habilidades algebraicas, trigonométricas y geométricas de los estudiantes en el curso de cálculo diferencial en ingeniería | *Measuring students’ algebra, trigonometry, and geometry skills on a differential calculus for engineering course. Revista Española de Pedagogía, 80* (282), 289-308. https://doi.org/10.22550/REP80-2-2022-07

https://revistadepedagogia.org/ ISSN: 0034-9461 (Impreso), 2174-0909 (Online)

culation of surfaces and volumes of geometric shapes. The present research provides teaching staff with important elements to adapt or modify their instructional designs and improve the learning quality of higher education students in the field of calculus. Additionally, secondary school teachers may benefit from these results regarding the greater challenges students face when enrolling in engineering programs.

Keywords: calculus, evaluation, reliability, measuring instrument

Resumen:

Se presentan los resultados de una investigación que incluyó la construcción y validación de un instrumento de medición para determinar las habilidades algebraicas, trigonométricas y geométricas que los estudiantes universitarios tienen al ingresar a una carrera de ingeniería y que son fundamentales para desempeñarse adecuadamente en los cursos de cálculo. En el diseño del instrumento participaron los profesores de la academia de matemáticas, todos con al menos grado de maestría y experiencia docente en el área de cálculo. El instrumento de medición quedó integrado por 40 reactivos y su análisis de calidad se describe y se deriva de las respuestas emitidas durante el ciclo lectivo 2020-2022 por 875 estudiantes de nuevo ingreso a la carrera de ingeniería. Los resultados muestran que los reactivos con dificultad media y con alta discriminación, son los que cuentan con mayor coeficiente de predicción y corresponden mayormente al área de geometría, específicamente en los temas de la línea recta, la circunferencia y el cálculo de superficies y volúmenes de figuras geométricas. Esta investigación aporta a los docentes elementos importantes para considerar ajustar o modificar sus diseños instruccionales y mejorar la calidad del aprendizaje de sus estudiantes universitarios en el campo del cálculo, así como también la consideración de los profesores del nivel medio respecto de las mayores dificultades que presentan los estudiantes que pretenden ingresar a los programas de ingeniería.

Descriptores: cálculo, evaluación, fiabilidad, instrumento de medida.

1. Introduction

The study of mathematics provides a very important foundation in engineering; it makes it possible to model different scientific phenomena and interpret and communicate in precise language (Brito et al., 2011). It also favours the development of abstract reasoning, which is fundamental when training engineers (Ruiz et al., 2016; Morales, 2009). The importance for engineering students of studying and understanding differential calculus lies in the formation of a solid conceptual platform, handling functions as mathematical models to represent quantitative and qualitative features, and acquiring knowledge of and the ability to apply a set of mathematical tools for solving science and engineering problems (Iglesias & Alonso, 2017; Ruiz et al., 2016).

Differential calculus is especially important on engineering degrees and is a prerequisite for courses such as integral calculus, differential equations, multivariable calculus, numerical methods, hydraulics, heat and mass transfer, statics, dynamics, electricity and magnetism, electrical circuits, and others.

In Mexico, standards of mathematics are a significant problem, as shown by the poor performance students from various subsystems have shown in standardised tests carried out nationally and internationally (Encinas et al., 2016). Academic performance, dropout, and repetition of courses are major problems, especially on degrees that require abstract logical thinking such as engineering. In addition, there is a lack of connection between the secondary education and higher education curricula, and the average grades are sometimes lower than the minimum pass mark for calculus courses (Hernández, 2005).

In recent decades there has been growing interest in studying the problem of academic failure, students not progressing to the next level, and abandonment by first-year degree students in engineering (Arraiz & Valecillos, 2010; Zavaleta & Flores, 2009; Correa et al., 2009), as well as in large-scale evaluation of learning, as they permit better knowledge and profiling of students' educational achievements. Evaluations make it possible to identify the skills acquired by students as a result of the teaching, and in turn make it possible to create strategies and assistance programmes to remedy low performance by students, which is a generalised concern in universities (Posso, 2005).

A previous study (Aguilar-Salinas et al., 2020) constructed and implemented a valid and reliable instrument for measuring the algebra skills that engineering students require to perform adequately on the differential calculus course and found that the most important shortcoming in students' algebra skills relates to the topic of rationalisation, division of polynomials, and factorising sums and differences of cubes.

In view of this, it was determined that university students require other types of skills to perform adequately on a differential calculus course in engineering degrees at the Universidad Autónoma de Baja California (UABC). Therefore, this research refers to the construction and validation of an instrument for measuring algebra, trigonometry, and geometry skills.

Establishing what level of algebra, trigonometry, and geometry skills these students acquired during their previous education is crucial when designing strategies to improve these skills and promote students' academic success and performance on the calculus for engineering courses.

2. Methodology

The research carried out is, on the one hand, a descriptive study as it is motivated by carrying out a detailed analysis of the technical quality of the items that make up the test. On the other hand, it is also an exploratory study by virtue of its possible findings with regards to the algebra, trigonometry, and geometry skills engineering students possess, which are a fundamental part of their performance on mathematics for engineering courses.

2.1. Method

To construct the measuring instrument, we adopted the model of Nitko (1994) for developing curriculum-driven exams. This model is complemented by the methodology for constructing criterion-referenced tests of Popham (1990) and with methodological and operational contributions from Contreras (2000). The analysis of the quality of the measuring instrument was done in accordance with Classical Test Theory (CTT), so that the instrument designed enables measurement of the algebra, trigonometry, and geometry skills required for successful completion of the calculus modules on an engineering degree. In view of the above, it is necessary to determine its reliability, content validity, and criterion validity, as well as its difficulty index, discrimination index, and biserial correlation (Carmines & Zeller, 1987).

The reliability analyses make measuring the consistency or stability of the measurements when the measurement process is repeated possible (Prieto & Delgado, 2010), thus determining their ability to display stability in their results (García & Vilanova, 2008). In this case, the Kuder-Richardson KR-20 coefficient and the split halves method were used. The reliability analysis using the Kuder-Richardson coefficient (KR-20) makes it possible to establish the reliability of an instrument based on the data obtained in a single application. The items are evaluated dichotomously and are considered to have different difficulty indexes (Corral, 2009). In the analysis of reliability by the split halves method, the test is divided in half (even and odd) and is separated into two parallel tests, and the internal consistency coefficient is used with the Spearman-Brown formula (Reidl-Martínez, 2013). If the instrument is reliable, there should be a strong correlation between the scores in the two halves.

In addition, the Ferguson delta coefficient was calculated, which measures the discriminating power of a complete test. The range of this coefficient is [0, 1] and it is satisfactory when it is greater than 0.90 (Ding et al., 2006).

The content validity was also calculated for the quality analysis of the instrument. This is established on the basis of suitable selection and indicators and is related to the mathematical processes and the testing of the validity of the items through expert judgement (Alsina & Coronata, 2014). In this type of validity test, a panel of experts with at least 5 years' experience in the topics being validated is selected who analyse the coherence of the items with what they set out to evaluate, the complexity of the items, and the cognitive ability to be evaluated (Barrazas, 2007) as well as the sufficiency and pertinence of the items. Here the aspects of the construct which are relevant, included in the competences and indicators, are considered (Cisneros et al., 2012).

Given that the measuring instrument designed here sets out to test students' command of knowledge that relates to algebra, trigonometry, and geometry content or topics that are regarded as necessary for studying and handling calculus in engineering programmes, a review was carried out with the aim of determining whether the items in the measuring instrument actually examine the topics and indicators of achievement established in the design specifications. This review was done by a panel of 5 university faculty members from the area of mathematics with a minimum of master's degree, who were not involved in the process of design and construction of the measuring instrument. The experts evaluated each of the 40 items from the measuring instrument, considering the parameters of pertinence, conceptual clarity, wording and terminology, scaling and codifying, and format. The choice of parameters and calculation of the content validity coefficient (CVC) were done in accordance with Hernández-Nieto (2002) and Gempp (2006) and also on the basis of the contributions of Urrutia et al. (2014) who recommend keeping items with a content validity coefficient equal to or greater than 0.80.

The criterion validity is determined through the correlation of the scores from applying the diagnostic measuring instrument studied here and the scores obtained with another external criterion (Hernández, Fernández & Baptista, 2006). In this case, the ordinary (final) grades that the students obtained in the differential calculus course in the 2020–2022 period were used as the external criterion.

The measuring instrument is a criteria-based test which sets out to measure skills in algebra, trigonometry, and geometry skills and so support the diagnosis of the instructional design for the calculus courses. The difficulty index (DI) is related to the proportion of students who correctly solve an item, and is calculated in accordance with Crocker and Algina (1986). There are parameters for accepting an item according to its level of difficulty. For CTT this index should be between 0.1 and 0.9. Backhoff et al. (2000) suggest that the values of the difficulty index should be distributed as follows: 5% very easy items ($0.87 < DI < 1$), 20% fairly easy ($0.74 < DI < 0.86$), 50% moderately difficult ($0.53 < DI < 0.73$), 20% fairly difficult ($0.33 < DI < 0.52$), and 5% very difficult ($DI < 0.32$).

The index of discrimination (IDC) of the item makes it possible to differentiate (discriminate) between students who obtained good marks on the test and one who obtained low marks. It is therefore related to a high likelihood that students who generally have very good performance on the test will correctly answer the item, and vice versa in the case of students with poor performance. In this analysis, 54% of population is considered, as 27% of the students with high performance are included as is an equal percentage of students with the lowest performance for each item reviewed.

Contreras and Backhoff (2004) and CTT, consider the discriminating power of the item appropriate if it is greater than 0.2. The scale of the IDC according to Backhoff et al. (2000) is: poor ($IDC < 0.20$), moderate ($0.20 < IDC < 0.30$), good ($0.30 < IDC < 0.40$), and very good ($D > 0.40$).

Another element considered to be important in the reliability and validity of the instrument relates to the correlation coefficient of the point biserial (rpbis), as this considers 100% of the population, not just 54% as in the case of the index of discrimination. According to Henrysson (1971), this coefficient is an indicator of predictive validity, in which a student's response to an item is related to the result the student obtains from the test. It is calculated in accordance with the model of Backhoff et al. (2000) and the scale of values for this indicator is: low discrimination (rpbis < 0.14), moderate (0.15 < rpbis < 0.25), good discriminatory power (0.26 < rpbis < 0.35), and excellent discrimination (rpbis > 0.35).

In addition, this analysis includes the development of item profiles. To this end, we used cluster analysis (Bausela, 2005; Castejón et al., 2016; Dixson et al., 2017; Gonçalves et al., 2017). This analysis is a type of data classification that is done by grouping the elements analysed. The fundamental objective of this type of analysis is to classify n objects into k (k > 1) groups, called clusters, by using p (p > 0) variables. The type of classification was k-means, as this is a tool designed to assign cases to a fixed number of groups.

The database was analysed using Classical Test Theory (CTT) and cluster analysis with the IBM SPSS Statistics 23 program and Excel spreadsheets, with which the psychometric data for each item, difficulty index, index of discrimination, correlation coefficients for the point biserial, and item profiles were obtained.

2.2. Process of creation of the measuring instrument

Six faculty members participated in the construction of the measuring instrument: two on the instrument design team, two on the specifications development team, and two on the items development team. All of the participating faculty members had a master's or doctorate, as well as at least five years' teaching experience in the fields of algebra, trigonometry, geometry, differential calculus, and integral calculus.

The function of the measuring instrument design team is to analyse the curriculum of the area, detect and structure content that it is important to evaluate, construct a table of specifications for the instrument and draw up a document explaining its decisions. The construction of the measuring instrument is based on the minimum algebra, trigonometry, and geometry skills that the engineering students need to perform well on the calculus courses on the engineering degree. These skills were determined by the instrument design team and validated by the specifications development team and by the panel of experts. The mathematical concepts and procedures involved in the measuring instrument form part of the curriculum that the students have followed throughout their studies on the programme of the mathematics module, both in the general baccalaureate and on the technological baccalaureate (SEP, 2017).

The purpose of the measuring instrument is to establish the initial conditions of the incoming students on an engineering degree with regards to the algebra, trigonometry, and geometry knowledge and skills required to take the calculus modules. To demonstrate these conditions, indicators of achievement were established for each specification and its respective item, which represent those traits of the student that make it possible to evaluate the degree of command of one of the skills described. The topics and indicators of achievement for each of the 40 items that comprise the measuring instrument are presented in Table 1.

Table 1. Topics and indicators of achievement for each item from the measuring.

Item Number	Topic Number	Topic	Indicator of Achievement
1	1	Exponential expressions	Multiplication of algebraic expressions, monomial by monomial, using the first law of exponents.
2	1	Exponential expressions	Multiplication of algebraic expressions, monomial by monomial, using the second law of exponents.
3	1	Exponential expressions	Division of algebraic expressions, monomial by monomial, using the fifth law of exponents.
4	1	Exponential expressions	Operations with radicals by rationalising the denominator or numerator.
5	1	Exponential expressions	Operations with radicals by rationalising the denominator or numerator.
6	1	Exponential expressions	Operations with radicals by rationalising the denominator or numerator.
7	2	Polynomials and special products	Simplifying algebraic expressions with like terms and grouping symbols.
8	2	Polynomials and special products	Multiplications with algebraic expressions, polynomial by polynomial.
9	2	Polynomials and special products	Divisions with algebraic expressions, polynomial by monomial.
10	2	Polynomials and special products	Divisions with algebraic expressions, polynomials by binomials.
11	2	Polynomials and special products	Multiplications with special products, squares of binomials.
12	2	Polynomials and special products	Calculating the multiplication of special products, conjugate binomials.
13	2	Polynomials and special products	Calculating the multiplication of special products, cubed binomials.

14	3	Factorising	Factorising using a common factor.
15	3	Factorising	Factorising difference of squares.
16	3	Factorising	Factorising with trinomials that are not perfect squares.
17	3	Factorising	Factorising sum of cubes.
18	3	Factorising	Factorising difference of cubes.
19	4	Rational expressions	Adding rational expressions.
20	4	Rational expressions	Multiplying rational expressions.
21	5	Trigonometry, angles and their measurement	Converting an angle from degrees to radians.
22	5	Trigonometry, angles and their measurement	Converting an angle from radians to degrees.
23	6	Trigonometric functions	Determining the value of the trigonometric function based on a right-angled triangle (sine, cosine, tangent).
24	6	Trigonometric functions	Determining the value of the trigonometric function based on a right-angled triangle (cosecant, secant, cotangent).
25	6	Trigonometric functions	Determining the value of the trigonometric function by constructing a right-angled triangle with special angles (sine, cosine, tangent).
26	6	Trigonometric functions	Determining the value of the inverse trigonometric function by constructing a right-angled triangle with special angles (arccotangent).
27	6	Trigonometric functions	Determining the value of the inverse trigonometric function by constructing a right-angled triangle with special angles (arccosine).
28	6	Trigonometric functions	Transforming a trigonometric function into its algebraic form.
29	6	Trigonometric functions	Resolving statements of problems by using right-angled triangles.

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30	6	Trigonometric functions	Resolving oblique triangles using the law of sines.
31	7	The straight line	Calculating the distance between two points from two given points.
32	7	The straight line	Calculating the slope of a straight line from two given points.
33	7	The straight line	Determining the general equation of a straight line based on one point and the slope.
34	7	The straight line	Determining the equation of a straight line from two points.
35	7	The straight line	Representing a linear equation algebraically based on its graphical representation.
36	8	Conic sections	Determining the graphical representation of a circumference from its algebraic representation.
37	8	Conic sections	Determining the algebraic representation of a circumference from its graphical representation.
38	9	Perimeter, area, and volume of geometric figures	Calculating the perimeter of a geometric figure.
39	9	Perimeter, area, and volume of geometric figures	Calculating the area of a geometric figure.
40	9	Perimeter, area, and volume of geometric figures	Calculating the volume of a geometric figure.

Source: Own elaboration.

The design of each of the 40 items is based on its respective specification (Graph 1), which considers aspects such as the algebra, trigonometry, or geometry topic to which it belongs, the indicator of achievement, according to Zabala and Arnau (2008), a comment about the meaning and functionality of the content, the basis of the item, the vocabulary and type of information that will be used in this item, the characteristics of the distractors, the process for obtaining the correct response, a sample item, and the estimated completion time. The design of the specification for each item was done by the specifications development team.

Graph 1. Specification corresponding to item 32 from the measuring instrument.

1. Identifying details of the content to be evaluated.			
		1.1 Item:	**21**
1.2 Subject: Geometry		**1.3 Macro Content:** The straight line.	
1.4 Topic: Slope of a straight line.	**1.5 Subtopic:** Calculating the slope of a straight line.		
2. Attributes of the item.			
2.1 Clarifying comment about the sense of the content The slope of a straight line in a rectangular system of representation (on a Cartesian plane), is usually represented by the letter *m*, and is defined as the difference on the Y axis divided by the difference on the X axis for two different points on a straight line. This is described in the following equation: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$			
2.2 Indicator of achievement	Calculating the slope of a straight line.		
2. Type of content	Concept ( )	Procedure (X)	
2.4 Difficulty	Reproduction (X)	Connection ( )	Reflection ( )
3. Attributes relevant to the stimuli that will be presented to the students.			
3.1 Instructions for responding to the item Select the option that corresponds to the correct value of the slope between two points.			
3.2 Basis of the item Two points are given on the Cartesian plane to calculate the slope between them using the slope formula.			
3.3 Vocabulary and textual, graphic or tabular information to use: The information provided in this item is textual and includes both points on the Cartesian plane, so that the student can calculate the slope between them.			
3.4 Distractors The following distractors are suggested for this item: a) Presenting an incorrect calculation. b) Modifying the result based on common mistakes in the formula.			
3.5 Correct response The correct slope between the two points.			
4. Sample item and time taken to resolve the item.			
4.1 Sample item Calculate the slope of the straight line that passes through the points (-5.1) and (1.4).			
A) $\frac{1}{2}$	B) 2	C) $-\frac{3}{4}$	D) $\frac{5}{4}$
4.2 Estimated time for completion 1 minute			

Source: Own elaboration.

The instrument comprises 40 items and is multiple choice as students are asked to choose the correct response from 4 possible options. Each item is independent, as they all contain the necessary information for stating and responding to it. The instrument is criteria-based, as its purpose is to evaluate learning by showing what the subject can and cannot do. The items were designed by the items development team based on the designed specifications.

3. Results and discussion

This section is divided into three parts: the first refers to the analysis of quality of the measuring instrument; the second part alludes to the analysis of the clusters of items; and the third and final one refers to the results students obtained in the diagnosis.

3.1. Analysis of the quality of the diagnostic measuring instrument

The measuring instrument was applied in the Mexicali Engineering Faculty (FIM) at the UABC during the first week of the 2020-2022 study cycle. The instrument was applied to 876 newly-enrolled students at the FIM who were taking the differential calculus course. The reliability of the instrument calculated using KR-20 is r = 0.95, and by the split halves method it is r = 0.93, which are considered to be appropriate when they are equal to or greater than 0.85 in the case of large-scale standardised instruments (Muñoz & Mato, 2008; Contreras & Backhoff, 2004). The distribution of the total scores was calculated using Ferguson's delta, giving a value of 0.99, which fully satisfies the criteria established (Engelhardt, 2009; Ding et al., 2006).

The average of the difficulty index was 0.68 ± 0.15 (mean ± standard deviation). The percentage distribution resulting from the DI is as follows: very easy items (2 items) 5%, fairly easy (17 items) 42.5%, moderate difficulty (13 items) 32.5%, fairly difficult (8 items) 20%, and very difficult (0 items) 0%. The minimum difficulty value was 0.34 while the maximum value is 0.89, both of which are acceptable in accordance with CTT and with a similar distribution to that proposed by Backhoff at el. (2000).

It is calculated that 75% of the items have excellent discrimination and 25% have good discrimination. The average discrimination index is 0.52 ± 0.13 (mean ± standard deviation), which is within a category considered as excellent (greater than 0.40). The minimum value with regards to discrimination was 0.31 and all of the items comply with this psychometric indicator (Contreras & Backhoff, 2004).

The average of the biserial correlation coefficients of the test is 0.49 ± 0.076 (mean ± standard deviation). It is calculated that 97.5% of the items have excellent discrimination, and 2.5% have good discrimination. No item was found with moderate, low, or negative discrimination.

With regards to the content validity, five experts took part and a CVC average of 0.89 ± 0.07 (mean ± standard deviation) was obtained with a minimum coefficient value of 0.82. The numbers above fully meet the criteria considered in this research for each item (Urrutia, Barrios, Gutiérrez & Mayorga, 2014; Gempp, 2006; Hernández-Nieto, 2002).

To determine the criterion validity, the ordinary grade that the students obtained on their differential calculus course during the 2020–2022 period of study was extracted from the records system of the department of academic services of the FIM. Of the 876 students who took the diagnostic test, we have records of the ordinary grades of 764 students for differential calculus, and we calculated the Pearson correlation between the score obtained in the diagnostic measuring instrument and the ordinary grade for differential calculus. When comparing the grades, a Pearson correlation coefficient of r = 0.313 was obtained. This correlation is significant at a level of 0.01 and so it is classed as a moderate correlation on the scale of Hernández et al. (2018). In other words, the higher the students' scores on the diagnostic measuring instrument for algebra, trigonometry, and geometry, the higher their ordinary grades on the differential calculus course. In addition, it was found that of the 764 students, 523 successfully completed the diagnostic instrument (a score equal to or greater than 60), of which 515 (98.5%) passed the differential calculus course.

3.2. Analysis of clusters of items

With the objective of establishing the significant features between psychometric indicators and the students' results, we carried out a k-means cluster analysis. The results were three profiles (Table 2) described below.

TABLE 2. Final cluster centres.

Psychometric indicators	Cluster		
	1	2	3
Difficulty index	0.81	0.46	0.62
Index of discrimination	0.42	0.47	0.65
rpbis	0.48	0.38	0.55
Number of items	17	6	17

Source: Own elaboration.

Cluster 1. This comprises 17 items, of which 70.6% are from algebra, 17.6% trigonometry, and 11.8% geometry. These are characterised by a higher difficulty index (0.81) and are classified as moderately easy. This group has the lowest indexes of discrimination (0.42) but have a fairly good predictive value (rpbis = 0.48). This shows that the algebra items (owing to their percentage value) have the lowest discrimination and are the easiest ones for the students to answer. This group mainly comprises items from the area of algebra relating to polynomials, special products, and exponential expressions, while in the area of trigonometry items referring to angles and their measurement are predominant.

Cluster 2. This comprises 6 items, of which 2 are from algebra and 4 from trigonometry. There are no geometry items. They are characterised by greater levels of difficulty (0.46). The discrimination (0.47) of the group is fairly acceptable and they have the lowest predictive value (0.38) although this is acceptable. This shows that the items from the trigonometry area are the ones that least predict students' success. The ones students find most difficult correspond to the application of the law of sines and solving problems through trigonometry of the right-angled triangle, while in the area of algebra, solving factorisation with difference of cubes and rationalising with rational expressions present students with the greatest difficulties.

Cluster 3. This comprises 17 items, of which 35.3% belong to the algebra area, 17.6% are from trigonometry, and 47% correspond to the geometry area.

The items in this group are characterised by being fairly difficult (0.62), by having a greater discrimination value (0.65), and by being items with the greatest prediction (0.55) compared to the rest of the clusters. It is apparent that the items that best predict students' success in the diagnostic measuring instrument are those from the area of geometry as 80% of the geometry items that make up the instrument are in this grouping. Straight lines, circumference, and calculating the perimeter, area and volume of geometric figures form part of the geometry subtopics with medium difficulty and the highest discrimination and prediction values.

3.3. Analysis of the results students obtained in the diagnostic instrument

In the first part of this analysis, the indicators of achievement and items that students have the most difficulties with solving correctly in the various areas that make up the measuring instrument were determined. In the second part, the difficulty by area of knowledge (algebra, trigonometry, and geometry) involved in the diagnostic instrument was established.

In the area of algebra: Item 6 (Graph 2), rationalising the numerator in an expression with a difficulty value of 0.34. It is considered that the difficulty of this item lies in the body of prior knowledge need and the application of rules to obtain the correct result, producing the conjugate, multiplying by the conjugate, and then simplifying are the series of steps that are normally required to rationalise an expression.

Item 10 (Graph 3), doing operations with algebraic expressions, polynomial by binomial, with a difficulty value of 0.46. The difficulty of this item is observed on the basis of the need to apply correctly the division algorithm (long procedure) and consider in this algorithm that the coefficient of the quadratic term is zero. Item 18, factorising with difference of cubes

Graph 2. Item 6 rationalising the numerator.

Question 6 2.5 points

What is the result of rationalising the numerator of the expression $\frac{\sqrt{4+x}-2}{x}$?

Choose at least one correct response.

A) $\frac{-1}{\sqrt{4+x}-2}$	
B) $\frac{x}{\sqrt{4+x}-2}$	
C) $\frac{x}{\sqrt{4+x}+2}$	
D) $\frac{1}{\sqrt{4+x}+2}$	*Correct response*

Source: Own elaboration.

GRAPH 3. Item 10, division of polynomial by binomial.

Question 10 2.5 points

What is the remainder from dividing $x^3 - 7x + 6$ by $x - 2$?

Choose at least one correct response.

A) 0	*Correct response*
B) 12	
C) -12	
D) 6	

Source: Own elaboration.

In the area of trigonometry, item 24 (DI = 0.37) and item 29 (DI = 0.41) stand out for their difficulty. The former refers to obtaining the value of the inverse trigonometric function of sine, cosine, or tangent of an angle, which initially involves fully calculating the right-angled triangle and then applying the definition of the trigonometric functions, specifically their reciprocals. The latter (Graph 4) relates to solving statements of problems using the right-angled triangle. Specialists have identified great difficulties for students in the case of problems from real life where solving them involves translation from natural language to algebra (Areaya & Sidelil, 2012).

GRAPH 4. Item 29, resolving statements of problems by using right-angled triangles.

Question 29 2.5 points

An escalator has an angle of 45º with the floor and lifts people by a vertical distance of 5 metres. If a person takes 20 seconds to go from the bottom of the escalator to the top, How fast is the escalator moving?

Choose at least one correct response.

A) $\sqrt{2}\, m/s$	
B) $4\, m/s$	
C) $\frac{4}{\sqrt{2}}\, m/s$	
D) $\frac{\sqrt{2}}{4}\, m/s$	*Correct response*

Source: Own elaboration.

In the field of geometry, item 39 (DI = 0.39) refers to calculating the area of a geometric figure (Graph 5) and is the most difficult for the students.

This is followed in difficulty by item 33 (DI = 0.47), which relates to determining the equation of the straight line based on a point and the slope (Graph 6). The difficulty of this item lies in the correct application of the point slope equation and of algebra skills, as the product, simplification, and finding the solution, given that it is an algorithm with a sequence of steps that is rather long for the student; the conventional path to reach the solution involves knowing and using the point slope equation, developing, simplifying, and making equal to zero.

GRAPH 5. Calculating the area of a geometric figure.

Question 39 2.5 points

The cylinder shown below has a height of $h = 5$ metres and a radius of $r = 2$ metres. What is the total surface area of cylinder?

$$h = 5$$
$$r = 2$$

The surface area S is 2 times the area of the base + the side area.

Choose at least one correct response.

A) $10\,\pi\,m^2$	
B) $20\,\pi\,m^2$	
C) $24\,\pi\,m^2$	
D) $28\,\pi\,m^2$	*Correct response*

Source: Own elaboration.

GRAPH 6. Determining the general equation of a straight line based on one point and the slope.

Question 33 2.5 points

What is the equation for the straight line that passes through point $P(-3,1)$ and has a slope of 2?

Choose at least one correct response.

A) $y - 2x - 7 = 0$	*Correct response*
B) $y - 2x - 5 = 0$	
C) $y + 2x - 7 = 0$	
D $y + 2x + 5 = 0$	

Source: Own elaboration.

In addition, the difficulty indexes by area of knowledge were calculated and no significant difference was found between the areas of trigonometry (DI = 0.66) and geometry (DI = 0.65), while the area of algebra was simpler for the students, as a difficulty of 0.72 was found, almost on the limit for classifying the area of algebra as fairly easy. This happens when the difficulty index is greater than 0.74 (Backhoff et al., 2000). Although the discrimination and prediction values are acceptable in all of the areas of knowledge, they are highest in geometry.

4. Conclusions

We constructed a valid and reliable instrument with the aim of determining the extent to which newly enrolled students on an engineering degree have the algebra, trigonometry, and geometry skills needed to take and successfully complete the differential calculus module, and of predicting the likelihood of success in this module.

A panel of experts evaluated whether the content of the items examined the proposed algebra, trigonometry, and geometry topics and whether the items are indicators of what they set out to measure. The judgements of the professionals were favourable in relation to the diagnostic possibilities of the measuring instrument. To determine the criterion validity, students' final grades from the differential calculus course were used as the criterion. Comparing the grades from the instrument with the criterion, gave a Pearson correlation coefficient $r = 0.313$, which is significant at the 0.01 level. Accordingly, high scores on the measuring instrument are translated into high grades on the differential calculus course, and successfully taking the diagnostic measuring instrument predicts that 98.5% of the students will go on to pass the differential calculus course. Consequently, this measuring instrument is regarded as a predictor of student performance on their differential calculus course for engineering degrees.

The reliability of the instrument calculated using KR-20 is $r = 0.95$, and by the split halves method it is $r = 0.93$. Therefore, the instrument is highly reliable and its use can be considered for large-scale application.

Rationalising rational expressions, dividing polynomials by binomials, solving problems that involve the trigonometry of right-angled triangles, calculating areas of geometric figures, and determining the general equation of a straight line are the topics that caused students the most problems in the diagnostic instrument and they follow the pattern that solving them requires prior knowledge and the application of successive rules.

Cluster analysis identified one cluster whose items better predict the success of students in the diagnostic instrument. In this cluster, items from the area of geometry predominate, the topics being: straight line, circumference, and calculating the perimeter area, and volume of geometric figures. The items from this cluster have medium difficulty and the highest discrimination values.

With the results from the application of this measuring instrument it is possible to identify the algebra, trigonometry, and geometry topics that students starting engineering degrees who will go on to take calculus modules find most difficult. At the same time, these results makes it possible to design the form and timing of the strategies needed to ensure that students have the algebra, trigonometry, and geometry skills required to complete the differential calculus module, as acquiring such skills directly affects students' academic performance.

The results of this research provide teachers with important elements to consider adjusting or modifying their instructional designs and improve the quality of their university students' learning in the field of calculus, as well as for secondary education teachers to consider regarding the greater difficulties presented by students who wish to enter engineering programmes.

References

Aguilar-Salinas, W. E., De Las Fuentes-Lara, M., Justo-López, A. C., & Martínez-Molina, A. D. (2020). Instrumento de medición para diagnosticar las habilidades algebraicas de los estudiantes en el Curso de Cálculo Diferencial en ingeniería [A measurement instrument for establishing the algebraic skills of engineering students on a Differential Calculus Course in engineering]. **revista española de pedagogía**, *78* (275). https://doi.org/10.22550/REP78-1-2020-02

Alsina, Á., & Coronata, C. (2014). Los procesos matemáticos en las prácticas docentes: diseño, construcción y validación de un instrumento de evaluación [Mathematical processes in teaching practices: design, construction and validation of an assessment instrument]. *Educación Matemática En La Infancia*, *3* (2), 23-36. http://www.edma0-6.es/index.php/edma0-6/article/view/129

Areaya, S., & Sidelil, A. (2012). Students' difficulties and misconceptions in learning concepts of limit, continuity and derivative. *The Ethiopian Journal of Education*, *32* (2), 1-38. http://213.55.95.79/index.php/EJE/article/view/343/246

Arraiz, G., & Valecillos, M. (2010). Regreso a las bases de la matemática: un imperativo en educación superior [Back to basics in mathematics: an imperative in higher education]. *Revista Digital Universitaria, 11* (9), 1-14. http://www.revista.unam.mx/vol.11/num9/art90/index.html

Backhoff, E., Larrazolo, N., & Rosas, M. (2000). Nivel de dificultad y poder de discriminación del examen de habilidades y conocimientos básicos (EXHCOBA) [The Difficulty Level and Discrimination Power of the Basic Knowledge and Skills Examination (EXHCOBA)]. *Revista Electrónica de Investigación Educativa*, *2* (1), 1-19. https://redie.uabc.mx/redie/article/view/15/26

Barrazas, A. (2007). La consulta a expertos como estrategia para la recolección de evidencias de validez basadas en contenido [Expert consultation as a strategy for collecting content-based validity evidence]. *Investigación Educativa Duranguense, 7*, 5-13. https://dialnet.unirioja.es/servlet/articulo?codigo=2358908

Bausela, E. (2005). SPSS: un instrumento de análisis de datos cuantitativos [SPSS: a quantitative data analysis tool]. *Revista de Informática Educativa y Medios Audiovisuales, 2* (4), 62-69. http://laboratorios.fi.uba.ar/lie/Revista/Articulos/020204/A3mar2005.pdf

Brito, M., Alemán, I., Fraga, E., Para, J., & Arias, R. (2011). Papel de la modelación matemática en la formación de los ingenieros [Role of mathematical modeling in the training of engineers]. *Ingeniería Mecánica, 14* (2), 129-139. http://scielo.sld.cu/pdf/im/v14n2/im05211.pdf

Carmines, E., & Zeller, R. (1987). *Reliability and validity.* Sage.

Castejón, J., Gilar, R., Miñano, P., & González, M. (2016). Latent class cluster analysis in exploring different profiles of gifted and talented students. *Learning and Individual Differences*, *50*, 166-174. https://doi.org/10.1016/j.lindif.2016.08.003

Cisneros, E., Jorquera, M., & Aguilar, Á. (2012). Validación de instrumentos de evaluación docente en el contexto de una universidad española [Validation of teaching evaluation instruments in the context of a Spanish university]. *Voces y Silencios: Revista Latinoamericana de Educación, 3* (1), 41-55. https://dialnet.unirioja.es/descarga/articulo/4054206.pdf

Contreras, L. (2000). *Desarrollo y pilotaje de un examen de español para la educación primaria en Baja California [Development and Piloting of a Spanish Language Test for Primary Education in Baja California]* [Master's tesis, Universidad Autónoma de Baja California]. Universidad Autónoma de Baja California Digital Archive. http://iide.ens.uabc.mx/documentos/divulgacion/tesis/MCE/1998/Luis_Angel_Contreras_Nino.pdf

Contreras, L., & Backhoff, E. (2004). Metodología para elaborar exámenes criteriales alineados al currículo [Methodology for developing curriculum-aligned criterion-referenced tests]. In Castañeda, S. (Eds.), *Educación aprendizaje y cognición, teoría en la práctica* (pp. 155-174). Manual Moderno.

Corral, Y. (2009). Validez y confiabilidad de los instrumentos de investigación para la recolección de datos [Validity and reliability of research data collection instruments]. *Revista Ciencias de la Educación*, *19* (33), 228-247. https://es.calameo.com/read/00441616680da9a5cd6ab

Correa, A., Chahar, B., Nieva, M., Figueroa, G., Gallo, R., & Holgado, L. (2009). Evaluando el rendimiento académico [Assessing academic performance]. *Acta Latinoamericana de Matemática Educativa, 22,* 317-326. http://funes.uniandes.edu.co/4759/1/CorreaEvaluandoAlme2009.pdf

Crocker, L., & Algina, J. (1986). *Introduction to classical and modern test theory.* Holt, Rinehart and Winston.

Ding, L., Chabay, R., Sherwood, B., & Beichner, R. (2006). Evaluating an electricity and magnetism assessment tool: Brief electricity and magnetism assessment. *Physical Review Special Topics - Physics Education Research, 2* (1). https://doi.org/10.1103/PhysRevSTPER.2.010105

Dixson, D., Worrell, F., & Mello, Z. (2017). Profiles of hope: How cluster of hope relate to school variables. *Learning and Individual Differences, 59*, 55-64. https://doi.org/10.1016/j.lindif.2017.08.011

Encinas, F., Osorio, M., Ansaldo, J., & Peralta, J. (2016). El cálculo y la importancia de los conocimientos previos en su aprendizaje [Calculus and the importance of prior knowledge in learning it]. *Revista de Sistemas y Gestión Educativa*, *3* (7), 32-41. https://www.ecorfan.org/bolivia/researchjournals/Sistemas_y_Gestion_Educativa/vol3num7/Revista_Sistemas_Gestion_Educativa_V3_N7_4.pdf

Engelhardt, P. (2009). An Introduction to classical test theory as applied to conceptual multiple-choice tests. *Getting Started in PER, 2* (1).

García, M., & Vilanova, S. (2008). Las representaciones sobre el aprendizaje de los alumnos de profesorado. Diseño y validación de un instrumento para analizar concepciones implícitas sobre el aprendizaje en profesores de matemática en formación [Learning representations of teaching students. Design and validation of an instrument to measure implicit learning conceptions in mathematics teaching students]. *Revista electrónica de investigación en educación en ciencias*, *3* (2), 27-35. https://ppct.caicyt.gov.ar/index.php/reiec/article/view/7409

Gempp, R. (2006). El error estándar de medida y la puntuación verdadera de los tests psicológicos: algunas recomendaciones prácticas [The standard error of measurement and the true score of psychological tests: Some practical recommendations]. *Terapia psicológica*, *24* (2), 117-129. https://www.redalyc.org/pdf/785/78524201.pdf

Gonçalves, T., Niemivirta, M., & Lemos, M. (2017). Identification of students' multiple achievement and social goal profiles and analysis of their stability and adaptability [Identificación de los perfiles de logros múltiples y objetivos sociales de los estudiantes y análisis de su estabilidad y adaptabilidad]. *Learning and Individual Differences, 54*, 149-159. https://doi.org/10.1016/j.lindif.2017.01.019

Henrysson, S. (1971). Gathering, analysing, and using data on test items. In R. L. Thorndike (Ed.), *Educational Measurement* (pp. 130-159). American Council on Education.

Hernández, A. (2005). El rendimiento académico de las matemáticas en alumnos universitarios [Academic performance in mathematicsin university students]. *Encuentro educacional*, *12* (1), 9-30. https://produccioncientificaluz.org/index.php/encuentro/article/view/861/863

Hernández, R., Fernández, C., & Baptista, P. (2006). *Metodología de la investigación [Research methodology].* Mc Graw Hill.

Hernández-Nieto, R. (2002). *Contributions to statistical analysis*. Universidad de Los Andes.

Hernández, J., Espinosa, J., Peñaloza, M., Rodríguez, J., Chacón, J., Toloza, C., Arenas, M., Carrillo, S., & Bermúdez, V. (2018). Sobre el uso adecuado del coeficiente de correlación de Pearson: definición, propiedades y suposiciones [On the proper use of the Pearson correlation coefficient: definitions, properties and assumptions]. *Archivos Venezolanos de Farmacología y Terapéutica*, *37* (5), 587-595. https://www.revistaavft.com/images/revistas/2018/avft_5_2018/25sobre_uso_adecuado_coeficiente.pdf

Iglesias, N., & Alonso I. (2017). Estudio exploratorio sobre la importancia de la matemática para la carrera de ingeniería civil en la Universidad de Oriente [Exploratory study about the importance of the mathematics for the career of civil engineering in the oriente university]. *REFCalE: Revista Electrónica de Formación y Calidad Educativa*, *5* (1), 45-62. http://refcale.uleam.edu.ec/index.php/refcale/article/view/1325/0

Morales, E. (2009). Los conocimientos previos y su importancia para la comprensión del lenguaje matemático en la educación superior [Background knowledge and its importance in mathematical language comprehension in higher education]. *Universidad, Ciencia y Tecnología, 13* (52), 211-222. http://ve.scielo.org/scielo.php?script=sci_arttext&pid=S1316-48212009000300004

Muñoz, J., & Mato, M. (2008). Análisis de las actitudes respecto a las matemáticas en alumnos de ESO [Analysis of attitudes towards mathematics in ESO students]. *Revista de Investigación Educativa*, *26* (1), 209-226. http://revistas.um.es/rie/article/view/94181

Nitko, A. (1994). *A model for developing curriculum-driven criterion-referenced and norm-referenced national examinations for certification and selection of students* [Paper presentation]. International Conference on Educational Evaluation and Assessment of the Association for the Study of Educational Evaluation in Southern Africa's (ASEESA) Pretoria, South Africa. https://eric.ed.gov/?id=ED377200

Posso, A. (2005). Sobre el bajo aprovechamiento en el curso de matemáticas I de la UTP [On the low achievement in the mathematics i course at utp]. *Scientia et Technica, 11* (28), 169-174.

Popham, J. (1990). *Modern educational measurement: A practitioner's perspective [La medición educativa moderna: la perspectiva de un profesional].* Allyn and Bacon, MA.

Prieto, G., & Delgado, A. (2010). Fiabilidad y validez [Reliability and validity]. *Papeles del Psicólogo*, *3* (1), 67-74. http://www.papelesdelpsicologo.es/pdf/1797.pdf

Reidl-Martínez, L. (2013). Confiabilidad en la medición [Reliability of measurement]. *Investigación en Educación Médica*, *2* (6), 107-111. http://www.scielo.org.mx/pdf/iem/v2n6/v2n6a7.pdf

Ruiz, E., Carmona, E., & Montiel, Á. (2016). Importancia del cálculo en el desarrollo académico del ingeniero [Importance of calculus in the academic development of the engineer]. *Pistas Educativas, 120*, 402-420. http://www.itcelaya.edu.mx/ojs/index.php/pistas/article/view/559

SEP (2017). *Planes de estudio de referencia del componente básico del marco curricular común de la educación media superior [Reference curricula of the basic component of the Common Curriculum Framework for Upper Secondary Education (CCEF)].* Secretaría de Educación Pública.

Urrutia, M., Barrios, S., Gutiérrez, M., & Mayorga, M. (2014). Métodos óptimos para determinar validez de contenido [Optimal method for content validity]. *Educación Médica Superior*, *28* (3), 547-558. http://scielo.sld.cu/pdf/ems/v28n3/ems14314.pdf

Zabala, A., & Arnau, L. (2008). *11 ideas clave. Cómo aprender y enseñar competencias [11 key ideas. How to learn and teach competences].* Editorial Grao.

Zavaleta, A., & Flores, C. (2009). Evaluación del currículum matemático escolar aprendido [Evaluation of the learned school mathematics curriculum]. *Red Cimates*, 702-712. https://core.ac.uk/download/pdf/322383805.pdf

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